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Phonon emission of two-dimensional plasmons

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Abstract. In this paper, we consider the acoustic phonon radiation by the plasma waves of quasi-two-dimensional inversion electron layers. The electron–phonon interaction is taken into account using the phenomenological deformation potential in the jellium model.

1. Introduction

A two-dimensional plasmon is a well known excitation of quasi-two-dimensional electrons on a He surface, in an inversion-channel Si metal–oxide–semiconductor field-effect transistor (MOSFET) or in GaAs–$Al_xGa_{1-x}As$ heterostructures (for a general review see Ando *et al* (1982)). The aim of this work is to investigate the phonon channel of the decay of the two-dimensional plasmon at frequencies below the optical frequencies of the crystal lattice. In spite of the fact that, for plasmon damping, the impurity scattering dominates, the plasmon-damping process may be important in such technical problems as the transformation of an electromagnetic field into a sound in the far-infrared (FIR) region.

2. Model

To investigate the qualitative properties of the effect, we restrict ourselves to the simplest possible model with only one filled surface subzone. Figure 1 shows a heterostructure of the type described in the work by Allen *et al* (1977) and Batke *et al* (1985). Electrons on a thin GaAs layer move in an almost free manner in the interface plane (x, y) while in the z direction they are localised by the potential barrier of a broader band $Al_xGa_{1-x}As$. In the flat-band model of Chen *et al* (1976), the electron wavefunction is given by the expression

$$\varphi_0(z) = (2/d)^{1/2}\sin(\pi z/d) \qquad 0 < z < d. \tag{1}$$

To generalise further considerations, we assume a complete set of wavefunctions φ_n describing both excited states of the GaAs layer shown in figure 1 and other layers in the case of a superlattice.

The frequency dependence of the conductivity is described well by the Drude formula

$$\sigma = (ne^2\tau/m)/(1 - \mathrm{i}\omega\tau) \tag{2}$$

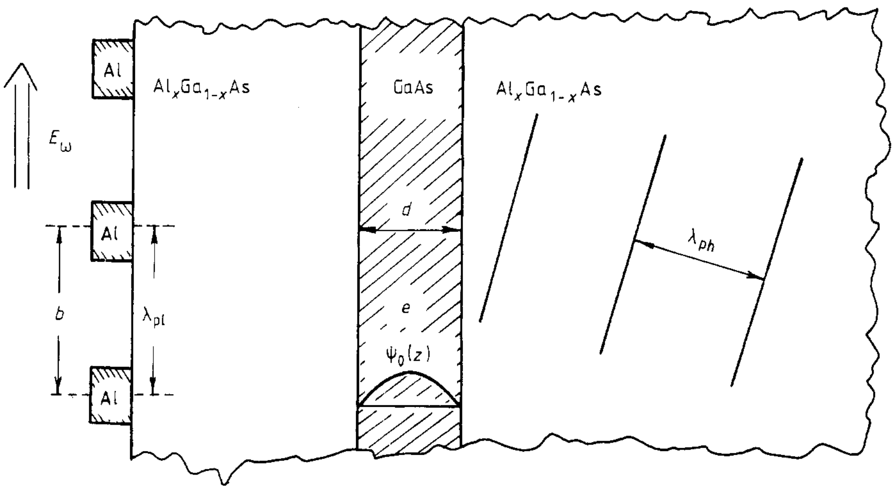

Figure 1. Transformation of the electromagnetic field into sound (schematically; not to scale). The microwave field $E\omega$ receives a momentum $\hbar/b$ from the Al grating and excites the plasmon which emits a phonon.

where e is the electron charge, n the number of electrons per unit area, m the effective electron mass ($m_{\mathrm{GaAs}} = 0.07$), ω the frequency and τ the relaxation time. For typical electron densities n of 10^{11}–10^{13} cm^{-2}, $\omega\tau = 1$ at $\omega = 10$–100 GHz (Koch 1976).

We shall take into account the electron interaction (except for the Coulomb interaction) with a crystalline lattice in the jellium model using the phenomenological deformation potential. In full analogy with superconductivity physics (for an introduction to this topic, see, e.g., Lifshitz and Pitaevskii (1982)) for the electron-electron interaction we have

$$U_{\omega,k} = (4\pi e^2/\varepsilon)/k^2 - gk^2/[k^2 - (\omega/u)^2 - \mathrm{i}0] \tag{3}$$

where ε is the dielectric constant (almost the same for GaAs and $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$), $\hbar k$ the transferred three-dimensional momentum, u the velocity of sound and g the coupling constant.

The coupling constant g is expressed in terms of the deformation potential Ξ_{d} and the crystal lattice density ρ:

$$g \simeq \Xi_{\mathrm{d}}^2/\rho u^2.$$

In the coordinate representation $\tau = (x, y, z)$ and in the static limit $\omega = 0$, we obtain

$$U(\tau) = (e^2/\varepsilon)/|\tau| - g\delta(\tau).$$

In the Bardeen–Cooper–Schrieffer theory, g parametrises the contact attraction between electrons caused by the virtual phonon exchange.

In a standard dielectric formalism, plasmon dispersion $\omega_{\mathrm{pl}}(q)$ is a solution of the dispersion equation

$$\det(\mathbf{I} - \mathbf{V}\Pi) = 0 \tag{4}$$

where $V_{\omega,q}$ are matrix elements of the interaction of quasi-two-dimensional electrons and $\Pi_{\omega,q}$ is a polarisation operator; the response of the two-dimensional electron density to small potential fluctuations is

$$\delta\varphi(\rho, t) = \varphi_{\omega,q} \exp(-\mathrm{i}\omega + \mathrm{i}q\rho)$$

where $q = (q_x, q_y)$ is a two-dimensional wave-vector $\rho = (x, y)$.

Wave damping arises both from the absorption part of the polarisation operator and from the imaginary addition of interaction matrix elements $V_{\omega,q}$ describing sound emission and escape of this phonon energy to the specimen bulk. Phonon damping will be obtained by separating out the term proportional to Ξ_d^2 from Im ω_pl.

Subsequently, in the following sections, we shall consider matrix elements of the interaction and the polarisation operator and, by analysing the plasmon dispersion, we shall obtain the intensity of phonon irradiation.

3. Matrix elements of interaction

The quantity $u_{\omega,k}$ defined by equation (3) is a three-dimensional propagator; to calculate the matrix elements $V_{nm,ij}$ of quasi-two-dimensional electrons, it is necessary to change to the coordinate representation in z direction. First, we shall calculate the matrix elements $W(z)$ of the interaction for two pure two-dimensional electrons (figure 2) moving in two parallel planes:

$$W(z) = \int \frac{U_{\omega,k} \exp(\mathrm{i}k_z z)}{2\pi} \, \mathrm{d}k_z$$

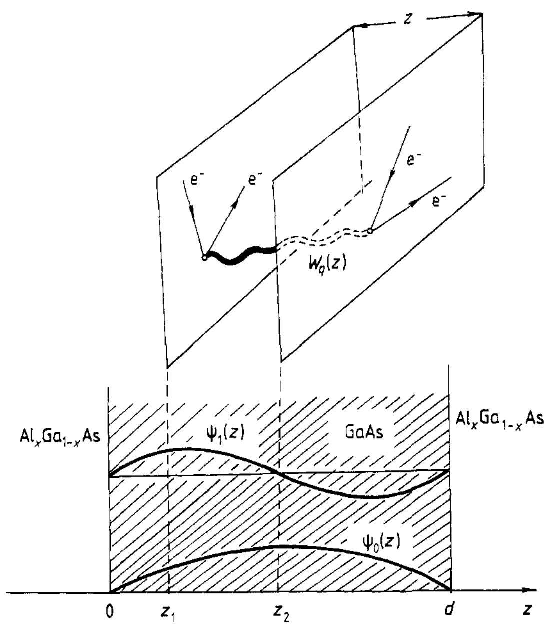

Figure 2. Calculation of the matrix elements V of interaction. The first problem is to calculate the scattering amplitude W for purely two-dimensional electrons moving in the planes z_1 and z_2. Then integration is to be made over localised φ functions in the z direction.

where z is the distance between the planes. Then, with the help of the wavefunctions $\varphi_n(z)$, we obtain the matrix elements participating in the dispersion relation (4):

$$V_{nm,ij} = \int \varphi_m^*(z_2)\varphi_n^*(z_2)W(z_2 - z_1)\varphi_i(z_1)\varphi_j(z_1)\,\mathrm{d}z_1\,\mathrm{d}z_2.$$

This formula, which we give for illustration purposes, shows that the matrix elements involve four-electron states. For our oversimplified model, only the diagonal term $i = j = m = n = 0$ with $\varphi_0(z)$ from equation (1) is relevant; of course, the corresponding matrix in equation (4) is 1×1.

Elementary integration gives

$$W(z) = (2\pi e^2/\varepsilon)\exp(-q|z|)/q - g[\delta(z) + q_0^2\exp(-Q|z|)/Q]$$

where $Q = (q^2 - q_0^2 - \mathrm{i}0)^{1/2}$, $q_0 = \omega/u$, $q_x = k_x$ and $q_y = k_y$.

A small imaginary addition io in the phonon propagator (3) determines the analytical continuation when the phase velocity ω/q is larger than the velocity u of sound:

$$Q \to -\mathrm{i}\tilde{Q} \qquad \tilde{Q} = (q_0^2 - q^2)^{1/2}.$$

For $\omega > qu$ the interaction $W(z)$, i.e. the amplitude of electron–electron scattering, receives an imaginary addition. The transition from the static phonon attraction to the emission of longitudinal acoustic phonons (see figure 1),

$$\exp(-Q|z|) \to \exp(\mathrm{i}Q|z|)$$

is analogous to the transition from a static Coulomb field to the Čerenkov irradiation of a superlight source in electrodynamics. Finally, for the matrix elements, we obtain (see figure 2)

$$\begin{aligned} V_{nm,ij} &= (2\pi e^2/\varepsilon q)\chi_{nm,ij}(q) - g[\kappa_{nm,ij} + \mathrm{i}(q_0^2/\tilde{Q})\chi_{nm,ij}(-\mathrm{i}\tilde{Q})] \\ \kappa_{nm,ij} &= \int \varphi_n(z)\varphi_m(z)\varphi_i(z)\varphi_j(z)\,\mathrm{d}z \qquad (5) \\ \chi_{nm,ij} &= \int \varphi_n(z_2)\varphi_m(z_2)\exp(-q|z_2 - z_1|)\,\varphi_i(z_1)\varphi_j(z_1)\,\mathrm{d}z_1\,\mathrm{d}z_2. \end{aligned}$$

Matrix elements of the Coulomb interaction (the first term in equation (5)) have many times been considered in the theory of different types of polariton in semiconductor superlattices (see, e.g., Das Sarma and Quinn (1982), and references therein). To avoid long expressions in the following analysis, let us make some simplifying assumptions. For all microstructures the grating period b (see figure 1) significantly exceeds the inversion layer thickness. For the Coulomb form factor $\chi(q)$, it is possible to use the long-wavelength limit $\chi_{00,00}(q) \simeq 1$, $qd \ll 1$. For two-dimensional plasmons, Mach's number Ma $= \omega/qu \gg 1$ and $\tilde{Q} \simeq q_0 = \omega/u$.

At low frequencies $q_0 d \ll 1$; in this case and for the phonon form factor, we have $\chi(-\mathrm{i}q_0) \simeq 1$. The latter equation is not always fulfilled. When the phonon wavelength $\lambda_{\mathrm{ph}} = 2\pi u/\omega \simeq d$ is comparable with the layer thickness, the destructive interference in the depth of the layer reduces the amplitude of phonon emission.

Under the above assumptions, to evaluate the order of magnitude of the phonon emission intensity, in the dispersion equation (4), we shall use

$$V \simeq (2\pi e^2/\varepsilon)/q - \mathrm{i}g(\omega/u). \qquad (6)$$

Let us now consider the response of the electron inversion layer to electric fields parallel to the interface.

4. Drude formula for the polarisation operator

For the two-dimensional electron density response

$$\delta n(\rho, t) = n_{\omega,q} \exp[\mathrm{i}(q\rho - \omega t)]$$

for small oscillations of the electrostatic potential given by

$$\delta\varphi(\rho, t) = \varphi_{\omega,q} \exp[\mathrm{i}(q\rho - \omega t)]$$

according to the definition (6), we have

$$n_{\omega,q} = \Pi_{\omega,q}(e\varphi_{\omega,q}).$$

If in Ohm's law with σ from equation (2)

$$j_{\omega,q} = \sigma_\omega E_{\omega,q}$$

we express the electric field in terms of the gradient of the potential, i.e.

$$E_{\omega,q} = -\mathrm{i}q\varphi_{\omega,q}$$

and use the charge conservation

$$qj_{\omega,q} = \omega(en_{\omega,q})$$

we obtain another representation of the Drude formula:

$$\Pi = (nq^2/m\omega^2)/(1 + \mathrm{i}/\omega\tau). \tag{7}$$

This is a long-wave fast asymptotic of the polarisation operator. For its application, it is necessary that the wavelength should be much larger than the inter-particle distance: $2\pi/q \gg n^{-1/2}$. Also, it is necessary that the phase velocity ω/q is much larger than the typical electron velocity, e.g. the Fermi velocity $v_F = \hbar n^{1/2}/m$ at low temperatures $k_B T \ll mv_F^2/2$. This form together with interaction (6) will be used in the dispersion equation (4).

5. Dispersion relation

Arbitrary oscillations of the electron density may be generated but, when as an intermediate state there are resonance excited plasmons, the intensity of the process may significantly increase. Under the wave propagation conditions $\omega\tau \gg 1$, and solving the dispersion equation $D = 1 - V\Pi = 0$ with Π from equation (7) and V from equation (6) in the zero-order approximation with respect to the small parameters g and τ^{-1}, we obtain the well known expression for plasmon dispersion:

$$\omega_{\mathrm{pl}}^2(q) = (2\pi ne^2/\varepsilon m)q.$$

The wavelength is fixed by the grating period $\lambda_{\mathrm{pl}} = 2\pi/q = b$.

Ohmic dissipation and phonon irradiation create small imaginary additions to the plasmon frequency $\omega = \omega_{\mathrm{pl}} - \mathrm{i}\gamma/2$. The imaginary part of the frequency is plasmon damping.

In the first approximation with respect to the small parameters g and τ^{-1} (as is accepted in plasma physics), we obtain

$$\gamma = 2\,\mathrm{Im}[D(\omega_{\mathrm{pl}}, q)]/\delta_{\omega}\,\mathrm{Re}[D(\omega, q)|_{\omega_{\mathrm{pl}}}]$$
$$= \omega_{\mathrm{pl}}\{(\omega_{\mathrm{pl}}\tau)^{-1} + g(\omega_{\mathrm{pl}}/u)\,\mathrm{Re}[\chi(-\mathrm{i}\omega_{\mathrm{pl}}/u)/(2\pi e^2/\varepsilon q)]\}$$
$$\mathrm{Re}[\chi_{mn,ij}(-\mathrm{i}\tilde{Q})] = C_{mn}C_{ij} - S_{mn}S_{ij}$$
$$C_{mn} = \int \varphi_m(z)\varphi_n(z)\cos(\tilde{Q}z)\,\mathrm{d}z$$
$$S_{ij} = \int \varphi_i(z)\varphi_j(z)\sin(\tilde{Q}z)\,\mathrm{d}z.$$

The first term, which is proportional to $(\omega\tau)^{-1}$ (always more important for γ), represents Ohmic dissipation. The second term, which is proportional to g, describes phonon emission. For the partial probability of the phonon decay, i.e. the ratio of the second to the first term, we obtain

$$P_{\mathrm{ph}} \simeq 2(\omega_{\mathrm{pl}}\tau)\{g/[(4\pi e^2/\varepsilon)/(q\omega_{\mathrm{pl}}/4)]\}\chi.$$

Comparison with superconductivity physics where the phonon term in interaction (3) is larger than the Coulomb term shows that the term $\{\ldots\} \sim 1$ if q and ω_{pl}/u are comparable with the reciprocal lattice constant. For the electron inversion layer, of course q, $\omega_{\mathrm{pl}}/u \ll 1$ Å^{-1}, which gives a rough estimate of the probability

$$P_{\mathrm{ph}} \simeq 1/\lambda_{\mathrm{pl}}\lambda_{\mathrm{ph}}.$$

where λ_{pl} and λ_{ph} are in ångströms. For a given probability of the phonon decay, the intensity of phonon irradiation may be evaluated from the intensity of generated plasmons. A more precise estimate for P_{ph} may be obtained by taking, for example,

$\Xi_{\mathrm{d}} = 16$ eV	$\rho = 5.3$ g cm^{-3}	$\varepsilon = 12.5$
$d = 260$ Å	$u = 5.0$ km s^{-1}	$n = 7 \times 10^{11}$ cm^{-2}
$b = 1\ \mu$m	$\tau = m\mu/e = 2 \times 10^{-12}$ s	$\mu = 5$ m^2 V^{-1} s^{-1}.

These parameters have been chosen using the work of Olego *et al* (1982) and *Landolt–Börnstein* (1982). Under these conditions

$P_{\mathrm{ph}} \simeq 10^{-5}$	$\omega_{\mathrm{pl}} = 6$ meV	Ma = 290
$\omega\tau = 17$	$q_0 d = 45$	$\lambda_{\mathrm{ph}} = 35$ Å

$\chi \simeq 1(q_0 d)^2$ for $q_0 d \gg 1$.

6. Discussion

The performed analysis is in agreement with the results of Krasheninnikov, Sultanov and Chaplik (KSC) (1979) who conclude that the phonon intensity, although several orders lower than that for piezocoupling, is within the possibility of contemporary experimental technique. The new phonon emission spectroscopy technique of a two-dimensional electron gas developed in the work of Rothenfusser *et al* (1986) is available.

It is important to mention that, owing to the grating, even black radiation generates resonant plasma waves with a fixed frequency and a parallel beam of monochromatic phonons. This may turn out to be important for some applications.

The increase in the electron mobility increases the probability of phonon decay $P \propto \tau$. A more interesting possibility for increase in the phonon intensity is provided in the case of superlattices, when N two-dimensional electron layers emit in phase (this will be the subject of a separate paper).

Finally it should be noted that the approach used is also suitable for estimating optical phonon generation when account is taken of the frequency dependence of the dielectric susceptibility

$$\varepsilon(\omega) = [(\omega^2 - \omega_{\mathrm{LO}}^2 - \mathrm{i}\gamma')/(\omega^2 - \omega_{\mathrm{TO}}^2 - \mathrm{i}\gamma')]\varepsilon_\infty.$$

For comparison see the paper by Das Sarma and Mason (1985).

7. Addendum

Finally we would like to draw attention to some qualitative properties in which our paper differs from that of KSC.

(i) In the paper by KSC the conclusion is drawn that emitted acoustic waves are almost shear. In our opinion, this is an inadequate peculiarity of the approximation used in their paper. Formally, plasmon attenuation caused by bulk phonon emission goes to zero when the transverse velocity of sound goes to zero. Thus, to emphasise this distinction, we used the jellium model in which only longitudinal compressing waves are taken into account.

(ii) In the paper by KSC, only the low-frequency case is considered, i.e. when the phonon wavelength is larger than the thickness of the electron layer. In our paper the corresponding form factor is introduced and the result is for the whole acoustic spectrum of phonon frequencies.

(iii) In the paper by KSC the boundary condition more appropriate for the free surface of a MOS structure is used whereas, in our work, conditions suitable for the two-dimensional electron gas buried in the bulk of one heterojunction are used.

The author is indebted to Professor Chaplik for clarifying this discussion.

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